Individual Project 7
Evaluation of Special Improper Integrals
Due Friday, Apr. 15

*****Read this first!** – You will need to email your professor (me) to get the specific integrals to use in your report. Be sure to do this early enough to get this data and complete your report. (e,g. if you email me at 11:00 PM, I probably will not get you the data until the following day.)

Project Assignment: This project is concerned with evaluating some improper integrals. In particular you will use an improper integral over an interval of infinite length to evaluate an integral of a function not defined at one end point. This will involve a special function Γ which arises in many applications of the sciences.

1. Evaluate the first integral assigned by your professor. Hint: use a substitution to simplify the problem and then use integration by parts.

2. Explain how you would evaluate the second integral assigned, but do not actually compute it. Would your method work for the third integral assigned? Why?

3. Define the function $\Gamma(t) = \int_0^\infty e^{-x}x^{t-1}\,dx$ for $t > 0$ and calculate $\Gamma(1)$.

4. Let t be a fixed number, $0 < t < 1$. Explain why $\Gamma(t)$ is an improper integral. Then show that the integral that defines $\Gamma(t)$ is convergent.

5. For $t > 0$, show that $\Gamma(t+1) = t\Gamma(t)$.

6. Let n be a positive integer. Use step 5 to find an equation relating $\Gamma(n)$ and $n!$. The factorial function is computed $n! = 1 \cdot 2 \cdot 3 \cdots n$. If you are not familiar with it, you may want to research it on the internet.

7. Let m be a positive real number and let n be a positive integer. Find an equation relating $\int_0^1 x^m(\ln x)^n\,dx$ to Γ and use this to find a formula for this integral in terms of m and n. (Hint: try the substitution $u = -(m+1)\ln x$.)

8. Use step 7 to evaluate the three integrals assigned by your professor.

You should print your report and turn it in by the due date.